

# Kerr-Bolt Spacetimes and Kerr/CFT Correspondence

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## Abstract

We investigate the recently proposed Kerr/CFT correspondence in the context of rotating spacetimes with a NUT twist. The Kerr/CFT correspondence states that the near-horizon states of an extremal four (or higher) dimensional black hole could be identified with a certain chiral conformal field theory. The corresponding Virasoro algebra is generated with a class of diffeomorphism which preserves an appropriate boundary condition on the near-horizon geometry. We try to understand the analog of singularities in the context of dual chiral CFT. Explicitly, we use Kerr/CFT correspondence to show that if we initially do not remove the singularities from the spacetimes, in the dual chiral CFT we can detect the presence of singularities in the bulk of spacetime.

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# 1 Introduction

For a long time, black holes have been an interesting theoretical system to understand the nature of quantum gravity. Despite a lot of efforts to explain and reproduce the Bekenstein-Hawking entropy, the theory of black hole entropy is not complete.

Recently, in the context of proposed Kerr/CFT correspondence [1], (see also [2] and [3]) the microscopic entropy of four-dimensional extremal Kerr black hole is calculated by studying the dual chiral conformal field theory associated with the diffeomorphisms of near horizon geometry of the Kerr black hole. These diffeomorphisms preserve an appropriate boundary condition at the infinity. One important feature of this correspondence is that it doesn't rely on supersymmetry and string theory unlike the well known AdS/CFT correspondence. [4, 5, 6, 7, 8, 9].

The Kerr/CFT correspondence has been used in [10] and [11] to find the entropy of dual CFT for four and higher dimensional Kerr black holes in AdS spacetimes and gauged supergravity as well as five-dimensional BMPV black holes in [12]. Moreover the correspondence has been used in string Theory D1-D5-P and BMPV black holes in [13] and in the five dimensional Kerr black hole in Gödel universe [14]. The continuous approach to the extremal Kerr black hole is essential in the proposed correspondence. For example, in the case of Reissner-Nordstrom black hole the approach to extremality is not continuous [15]. The rotating bubbles, Kerr-Newman black holes in (A)dS spacetimes and rotating NS5 branes have been considered in [16], [17] and [18]. The four-dimensional Kerr-Sen black hole has been considered in [19].

Inspired with these works, in this article, we consider the class of Kerr-Bolt spacetimes. The Kerr-Bolt spacetimes are exact solutions to the four-dimensional gravity. The spacetimes with NUT twist have been studied extensively in regard to their conserved charges, maximal mass conjecture and D-bound in [20]. These spacetimes have conical and Dirac-Misner singularities that should be removed by identifications of coordinates in the metric.

We apply the Kerr/CFT correspondence to the extremal Kerr-Bolt spacetimes (these extremal spacetime exist if we don't remove the singularities from Kerr-Bolt spacetimes) and conclude that in the context of Kerr/CFT correspondence, the results from CFT side can show and detect the presence of singularities in the bulk. The results of this paper are all in favor of Kerr/CFT correspondence.

The outline of this paper is as follows. In section 2, we first review briefly the rotating spacetimes with the NUT charge and show that the only consistent rotating spacetimes with the NUT charge are Kerr-Bolt spacetimes where the fixed point set of the Killing vector is two dimensional sphere. In section 3, we find the near-horizon geometry of extremal spacetimes by using a special coordinate transformations. In section 4, we calculate the central charge and microscopic entropy of the extremal Kerr-Bolt spacetimes. We then study the first law of thermodynamics in CFT and find that the first law of thermodynamics cannot be satisfied except if the spacetime is free of any NUT charge. This in turn, indicates that the NUT charges induce singularities in the spacetime. In section 5, we review how we should define a positive definite and consistent expression with the first law, for the entropy of non-extremal

Kerr-Bolt spacetimes. We conclude in section 6 with a summary of our results.

## 2 Kerr-Bolt Spacetimes

In this section, we give a brief review of the Kerr-Bolt spacetime. The Lorentzian geometry of Kerr spacetimes with NUT charge and nonzero cosmological constant is given by the line element

$$ds^2 = -\frac{\Delta_L(r)}{\Xi_L^2 \rho_L^2} [dt + (2n \cos \theta - a \sin^2 \theta) d\varphi]^2 + \frac{\Theta_L(\theta) \sin^2 \theta}{\Xi_L^2 \rho_L^2} [adt - (r^2 + n^2 + a^2) d\varphi]^2 + \frac{\rho_L^2 dr^2}{\Delta_L(r)} + \frac{\rho_L^2 d\theta^2}{\Theta_L(\theta)} \quad (2.1)$$

where

$$\begin{aligned} \rho_L^2 &= r^2 + (n + a \cos \theta)^2 \\ \Delta_L(r) &= -\frac{r^2(r^2 + 6n^2 + a^2)}{\ell^2} + r^2 - 2mr - \frac{(3n^2 - \ell^2)(a^2 - n^2)}{\ell^2} \\ \Theta_L(\theta) &= 1 + \frac{a \cos \theta (4n + a \cos \theta)}{\ell^2} \\ \Xi_L &= 1 + \frac{a^2}{\ell^2} \end{aligned} \quad (2.2)$$

which are exact solutions of the Einstein equations. The “event horizons” of the spacetime are given by the singularities of the metric function which are the real roots of  $\Delta_L(r) = 0$ . These are determined by the solutions of the equation

$$r_+^4 - r_+^2(\ell^2 - 6n^2 - a^2) + 2m\ell^2 r_+ + (3n^2 - \ell^2)(a^2 - n^2) = 0 \quad (2.3)$$

The “Euclidean section” for this class of metrics is given by

$$ds^2 = \frac{\Delta_E(r)}{\Xi_E^2 \rho_E^2} [dt - (2n \cos \theta - a \sin^2 \theta) d\varphi]^2 + \frac{\Theta_E(\theta) \sin^2 \theta}{\Xi_E^2 \rho_E^2} [adt - (r^2 - n^2 - a^2) d\varphi]^2 + \frac{\rho_E^2 dr^2}{\Delta_E(r)} + \frac{\rho_E^2 d\theta^2}{\Theta_E(\theta)} \quad (2.4)$$

where we have analytically continued the  $t$  coordinate, the nut charge and the rotation parameter to imaginary values, yielding

$$\begin{aligned} \rho_E^2 &= r^2 - (n + a \cos \theta)^2 \\ \Delta_E(r) &= -\frac{r^2(r^2 - 6n^2 - a^2)}{\ell^2} + r^2 - 2mr - \frac{(3n^2 + \ell^2)(a^2 - n^2)}{\ell^2} \\ \Theta_E(\theta) &= 1 - \frac{a \cos \theta (4n + a \cos \theta)}{\ell^2} \\ \Xi_E &= 1 - \frac{a^2}{\ell^2} \end{aligned} \quad (2.5)$$

The Euclidean section exists only for values of  $r$  such that the function  $\Delta_E(r)$  is positive valued. The horizons are located at the zeros of  $\Delta_E(r)$ , which we shall denote by  $r = r_0$ . Moreover, the range of  $\theta$  depends strongly on the values of the NUT charge  $n$ , the rotational parameter  $a$  and the cosmological constant  $\Lambda = 3/\ell^2$ , taken here to be positive (for the solution with  $\Lambda < 0$ , replace  $\ell^2$  with  $-\ell^2$  in the preceding equations). The angular velocity of the horizon in the Lorentzian geometry is given by

$$\Omega_H = -\frac{g_{t\varphi}}{g_{\varphi\varphi}} \Big|_{r=r_0} = \frac{a}{r_0^2 + n^2 + a^2} \quad (2.6)$$

and so the surface gravity of the cosmological horizon can be calculated to give

$$\kappa = \frac{1}{2(r_0^2 + n^2 + a^2)\Xi_L} \left. \frac{d\Delta_L}{dr} \right|_{r=r_0} \quad (2.7)$$

where the Killing vector  $\chi^\mu = \zeta^\mu + \Omega_H \psi^\mu$  is normal to the horizon surface  $r = r_0$ .

We first note that there are no pure NUT solutions for nonzero values of  $a$ . We demonstrate this as follows.<sup>2</sup> Since  $\psi^\mu$  is a Killing vector, for any constant  $\varphi$ -slice near the horizon, additional conical singularities will be introduced in the  $(t, r)$  section unless the period of  $t$  is  $\Delta t = \frac{2\pi}{|\kappa|}$ . Furthermore, there are string-like singularities along the  $\theta = 0$  and  $\theta = \pi$  axes for general values of the parameters. These can be removed by making distinct shifts of the coordinate  $t$  in the  $\varphi$  direction near each of these locations. These must be geometrically compatible [21], yielding the requirement that the period of  $t$  should be  $\Delta t = 4n\Delta\phi = \frac{16\pi n}{q_+ + q_-}$ . Demanding the absence of both conical and Dirac-Misner singularities, we get the relation

$$\frac{k}{\kappa} = \frac{8n}{q_+ + q_-} \quad (2.8)$$

where  $k$  is any non-zero positive integer. Demanding the existence of a pure NUT solution at  $r = r_0$  is equivalent to the requirement that the area of the surface of the fixed point set of the Killing vector  $\partial/\partial t$ ,

$$A = \frac{2\pi}{\Xi_E} \{ \alpha \sqrt{\Delta_E(r_0)} + 2(r_0^2 - n^2 - a^2) \} \quad (2.9)$$

vanish, where  $\alpha = \int_0^\pi \frac{2n \cos \theta - A \sin^2 \theta}{\sqrt{\Theta_E(\theta)}} d\theta$ . In other words, this surface is of zero dimension. This can only occur special values of the mass parameter. However if we select for this parameter we find an inconsistency with the relation (2.8), which must hold for the spacetime with NUT charge.

Hence we conclude that the only spacetimes with NUT charge and rotation is Taub-Bolt-Kerr-(A)dS spacetimes (or simply Kerr-Bolt-(A)dS), where the term ‘‘bolt’’ refers to the fact that the dimensionality of the fixed point set of  $\partial/\partial t$  is two.

In this article, we consider Kerr/CFT correspondence in the asymptotically flat case and leave the cases with cosmological constant for further investigations.

### 3 Near-Horizon Geometry

In this section, we study the near-horizon geometry of Kerr-Bolt spacetimes without any constraint as (2.8). The Kerr-Bolt spacetime is given by the line element

$$ds^2 = -\frac{\Delta(\tilde{r})}{\rho^2} [dt + (2n \cos \theta - a \sin^2 \theta) d\tilde{\phi}]^2 + \frac{\sin^2 \theta}{\rho^2} [a d\tilde{t} - (\tilde{r}^2 + n^2 + a^2) d\tilde{\phi}]^2 + \frac{\rho dr^2}{\Delta(\tilde{r})} + \rho^2 d\theta^2 \quad (3.1)$$

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<sup>2</sup>It also follows purely topologically from the fact that the surfaces of constant  $r$  are not homeomorphic to  $S^3$ .

where

$$\rho^2 = \tilde{r}^2 + (n + a \cos \theta)^2 \quad (3.2)$$

$$\Delta(r) = \tilde{r}^2 - 2m\tilde{r} + a^2 - n^2 \quad (3.3)$$

The event horizon is located at  $r_0 = m + \sqrt{m^2 - a^2 + n^2}$  with angular velocity (2.6). The Hawking temperature is equal to

$$T_H = \frac{\sqrt{m^2 - a^2 + n^2}}{2\pi(r_0^2 + n^2 + a^2)} \quad (3.4)$$

To find the near-horizon limit of the extreme black hole, we change the coordinates by the following transformations

$$\tilde{r} = m(1 + \frac{\lambda}{y}) \quad (3.5)$$

$$\tilde{t} = \frac{2a^2}{r_0\lambda}t \quad (3.6)$$

$$\tilde{\phi} = \phi + \frac{a}{r_0}t/\lambda \quad (3.7)$$

where the scaling parameter  $\lambda$  approaches zero. The metric (3.1) changes then to the near-horizon metric given by

$$\begin{aligned} ds^2 &= \{a^2(1 + \cos^2(\theta)) + 2na \cos(\theta)\} \left\{ \frac{-dt^2 + dy^2}{y^2} + d\theta^2 \right\} \\ &+ \frac{4a \sin^2(\theta)}{a(1 + \cos^2(\theta)) + 2n \cos(\theta)} (ad\phi + \sqrt{a^2 - n^2} \frac{dt}{y})^2 \end{aligned} \quad (3.8)$$

The metric definitely is not asymptotically flat. To cover the whole near-horizon geometry, we use the global coordinates

$$y = \frac{1}{\cos \tau \sqrt{1 + r^2} + r} \quad (3.9)$$

$$t = y \sin \tau \sqrt{1 + r^2} \quad (3.10)$$

$$\phi = \frac{\sqrt{a^2 - n^2}}{a} \left\{ \varphi + \ln \left( \frac{\cos \tau + r \sin \tau}{1 + \sin \tau \sqrt{1 + r^2}} \right) \right\} \quad (3.11)$$

The global near-horizon metric is

$$\begin{aligned} ds^2 &= \{a^2(1 + \cos^2(\theta)) + 2na \cos(\theta)\} \left\{ -(1 + r^2)d\tau^2 + \frac{dr^2}{1 + r^2} + d\theta^2 \right\} \\ &+ \frac{4 \sin^2(\theta)}{\{a(1 + \cos^2(\theta)) + 2n \cos(\theta)\}^2} (a^2 - n^2)(d\varphi + rd\tau)^2 \end{aligned} \quad (3.12)$$

In the case of vanishing NUT charge, the metric becomes the near-horizon geometry of the Kerr solution, as in [1, 22]. For a fixed  $\theta$ , the near-horizon geometry is a quotient of warped  $\text{AdS}_3$  which the quotient arises from identification of  $\varphi$  coordinate. The isometry group of the geometry is  $SL(2, R) \times U(1)$ , where  $U(1)$  is generated by the Killing vector  $-\partial_\varphi$  and  $SL(2, R)$  is generated by three Killing vectors,

$$J_1 = 2 \sin \tau \frac{r}{\sqrt{1+r^2}} \partial_\tau - 2 \cos \tau \sqrt{1+r^2} \partial_r + \frac{2 \sin \tau}{\sqrt{1+r^2}} \partial_\varphi \quad (3.13)$$

$$J_2 = -2 \cos \tau \frac{r}{\sqrt{1+r^2}} \partial_\tau - 2 \sin \tau \sqrt{1+r^2} \partial_r - \frac{2 \cos \tau}{\sqrt{1+r^2}} \partial_\varphi \quad (3.14)$$

$$J_3 = 2 \partial_\tau \quad (3.15)$$

## 4 Microscopic Entropy in CFT and Failure of First Law

Choosing the proper boundary condition for the near-horizon metric as the same as one in [1], it can be shown that the near-horizon metric has a class of commuting diffeomorphisms labeled by  $p = 0, \pm 1, \pm 2, \dots$

$$\zeta_p = -e^{-ip\varphi} (\partial_\varphi + ipr \partial_r) \quad (4.1)$$

This diffeomorphism generates a Virasoro algebra without any central charge

$$[\zeta_p, \zeta_q] = -i(p-q)\zeta_{p+q} \quad (4.2)$$

The generator of  $\zeta_p$  is the conserved charge  $Q_\zeta[g]$  between neighboring geometries  $g$  (which is the near-horizon metric (3.12)) and  $h$  (which is the deviation from the near-horizon metric (3.12)) and is given by

$$Q_{\zeta_p}[g] = \frac{1}{8\pi} \int_{\partial\Sigma} k_{\zeta_p}[h, g] \quad (4.3)$$

In above equation,  $\partial\Sigma$  is the boundary of a spatial slice and two form  $k_\zeta$  is

$$\begin{aligned} k_\zeta[h, g] &= -\frac{1}{4} \epsilon_{\mu\nu\rho\sigma} \{ \zeta^\sigma \nabla^\rho h - \zeta^\sigma \nabla_\lambda h^{\rho\lambda} + \zeta_\lambda \nabla^\sigma h^{\rho\lambda} + \frac{1}{2} h \nabla^\sigma \zeta^\rho \\ &\quad - h^{\sigma\lambda} \nabla_\lambda \zeta^\rho + \frac{1}{2} h^{\lambda\sigma} (\nabla^\rho \zeta_\lambda + \nabla_\lambda \zeta^\rho) \} dx^\mu \wedge dx^\nu \end{aligned} \quad (4.4)$$

The Dirac brackets of two conserved charges  $Q_{\zeta_p}$  and  $Q_{\zeta_q}$  yields  $Q_{[\zeta_p, \zeta_q]}$  and a central term which is equal to  $\frac{1}{8\pi} \int_{\partial\Sigma} k_{\zeta_p}[\mathcal{L}_{\zeta_q} g, g]$ , where  $\mathcal{L}_{\zeta_q} g$  is the Lie derivative of the near-horizon metric (3.12). Replacing the Dirac brackets by commutators and straightforward calculation of the central term yields a Virasoro algebra with the central charge

$$c = 12a\sqrt{a^2 - n^2} \quad (4.5)$$

for the dual chiral CFT corresponding to Kerr-bolt spacetime (3.8). To find the entropy of dual chiral CFT, we need to find Frolov-Thorne temperature [23]. A straightforward calculation shows the right temperature  $T_R = 0$  and the left temperature

$$T_L = \frac{a}{2\pi\sqrt{a^2 - n^2}} \quad (4.6)$$

hence the Frolov-Thorne temperature is  $T_{FT} = T_L$ . Finally, we get the microscopic entropy in dual chiral CFT by using the Cardy relation

$$S = \frac{\pi^2}{3} c T_{FT} = 2\pi a^2 = 2\pi(m^2 + n^2) \quad (4.7)$$

In the special case of  $n = 0$ , this result is exactly in agreement with the macroscopic Bekenstein-Hawking entropy of Kerr black hole [1]. On the other hand for non-zero NUT charge  $n$ , the microscopic result (4.7) does not satisfy the microscopic first law of thermodynamics. In fact, we show the first law of thermodynamics gives for the microscopic temperature something quite different from Frolov-Thorne temperature given by (4.6). To see this, we consider the first law as

$$TdS = dM - \Omega_H dJ \quad (4.8)$$

Here we consider the entropy as a function of  $M = m$  and  $J = ma$  where we choose  $q_+ = q_- = 1$  for simplicity. (see section (5) for details). By introducing a parameter  $\delta = m - \sqrt{a^2 - n^2}$  which measures non-extremality, the entropy can be considered as a function of  $\delta$  and  $J$ . So the first law can be rewritten as

$$dS = \beta_H d\delta + \beta dJ \quad (4.9)$$

where  $\beta = (\frac{\partial S}{\partial J})_{\text{fixed } \delta}$ . A straightforward calculation shows

$$\beta = \beta_H \left\{ \frac{2J}{\sqrt{n^4 + 4J^2} \sqrt{-2n^2 + 2\sqrt{n^4 + 4J^2}}} - \frac{1}{\sqrt{2n^2 + 2\sqrt{n^4 + 4J^2}}} \right\} \quad (4.10)$$

where  $\beta_H = 1/T_H$ . This equation gives the temperature of chiral CFT as  $T = 0$ , not in agreement with (4.6). We note that the only consistent result from equation (4.10) with Frolov-Thorne temperature (4.6), is in the special case of  $n = 0$ . In this case (4.10) reproduces exactly  $T = \frac{1}{2\pi}$  in agreement with the temperature of dual CFT to Kerr black hole. So we conclude applying Kerr/CFT correspondence to the metric (2.1) without removing its conical and Misner-Dirac singularities (given by equation (2.8)), leads to violation of the microscopic first law in CFT side.

## 5 Entropy of non-extremal Kerr-Bolt Spacetimes

In this section, we discuss briefly about the entropy of non-extremal Kerr-Bolt spacetimes in both Euclidean and Lorentzian signatures. Although the entropy of non-extremal black

holes was discussed in literature [24], but it can be shown that the results don't satisfy the first law of thermodynamics. The reason for this (as we see later in this section) is that the singularities on the north and south poles as well as on horizon should be removed consistently. This leads to introducing three vectors that generate a lattice and they should be minimal and commensurable (see equations (5.3), (5.4), (5.5), (5.8)).

All the instantons we will consider will be derived from Kerr-Bolt line element by identifications. As we will see, these identifications will not only twist the imaginary-time direction near infinity, but also modify even the topology of the “spatial” two-sphere at infinity, somewhat in the manner of the so called “Asymptotically Locally Euclidean instantons”. That is, the topology at infinity will (unfortunately) not necessarily be that of a circle bundle over  $S^2$ .

The line element is singular along the polar axes  $\theta = 0, \pi$  and at the “horizon”  $r = r_0$ . The “string” singularities at the poles have, as is well known, a more complicated structure than that belonging to the familiar spherical coordinates for  $S^2$ , and they cannot be removed merely by the imposition of periodicity in azimuth  $\varphi$ . Rather two separate identifications are required to remove the two “strings”. Moreover, one needs a further identification to remove the singularity at the horizon, and all of the identifications must be compatible. This compatibility condition is crucial, but it seems to have been overlooked in the literature on these metrics. To analyze it, we may refer everything to a lattice in the time-azimuth plane.

The vectors  $\partial/\partial t$  and  $\partial/\partial\varphi$  are both Killing vectors so this makes it possible to quotient our metric by any linear combination of the two (with constant coefficients). More generally, we can quotient by any *lattice*  $\mathcal{L}$  of vectors which is closed under vector sum and difference. The resulting spacetime is entirely determined by  $\mathcal{L}$ . Let us consider how  $\mathcal{L}$  must be chosen in order to remove the two types of singularity we have to deal with.

To take advantage of the scale invariance of our family of metrics, we will introduce in place of  $t$  the dimensionless coordinate

$$\psi = t/2n , \tag{5.1}$$

and the corresponding Killing vector

$$\partial/\partial\psi = 2n \partial/\partial t . \tag{5.2}$$

Now look at the neighborhood of the north polar axis,  $\theta = 0$ . At the pole, the line-element acquires a degenerate direction. In order to compensate, we must quotient by a vector  $\eta_+$  that is parallel to this degenerate direction and whose length is chosen so that the quotient metric will not exhibit a conical singularity. That is, the length of  $\eta_+$  at a proper distance  $\varepsilon$  from the north pole, must be  $2\pi\varepsilon$  to first order in  $\varepsilon$ . Some algebra shows that the required vector is [21]

$$\eta_+ = 2\pi (\partial/\partial\psi + \partial/\partial\varphi) . \tag{5.3}$$

This therefore is one of the vectors of  $\mathcal{L}$ , but we can say more. It is also necessary that no “submultiple” of  $\eta_+$  belong to  $\mathcal{L}$ ; otherwise we would really be quotienting by a smaller vector. That is,  $\eta_+$  must be a “minimal” element of the lattice. (By minimal we mean that



$\lambda\eta_+ \notin \mathcal{L}$  for  $0 < \lambda < 1$ .) The same analysis at the south pole furnishes a second minimal lattice vector,

$$\eta_- = 2\pi(\partial/\partial\psi - \partial/\partial\varphi) . \quad (5.4)$$

The singularity at  $r = r_0$  can be analyzed similarly. Although the algebra is a bit messier, it proceeds along the same lines and reveals a third minimal lattice vector,

$$\xi = \beta_H(\partial/\partial t + \Omega_H\partial/\partial\varphi) , \quad (5.5)$$

where  $\beta_H$  and  $\Omega_H$  are given by

$$\beta_H = \frac{4\pi(r_0^2 - n^2 - a^2)r_0}{r_0^2 - n^2 + a^2} \quad (5.6)$$

and

$$\Omega_H = \frac{a}{r_0^2 - n^2 - a^2} \quad (5.7)$$

Finally, there are the special points where the polar axes meet the horizon. When the angular velocity vanishes, no extra conditions on  $\mathcal{L}$  are imposed by regularity at these special points; by analyticity there is no reason to suppose that extra conditions arise when rotation is “turned on”.

What does impose a crucial extra condition, however, is the requirement that our quotient spacetime be a manifold. If the lattice  $\mathcal{L}$  were dense in the  $\varphi$ - $t$ -plane, this obviously would not be the case, as the quotient  $\mathbb{R}^2/\mathcal{L}$  would be pathologically non-Hausdorff. In order to preclude this, one must arrange that three vectors  $\eta_\pm$  and  $\xi$  be *commensurate*. (In other words,  $\xi$  must be a linear combination of  $\eta_\pm$  with rational coefficients.) Bringing in the minimality constraints as well, one can see that the complete condition is

$$p\xi = q_+\eta_+ + q_-\eta_- , \quad (5.8)$$

where  $p$ ,  $q_+$  and  $q_-$  are three relatively prime integers:  $(p, q_+) = (p, q_-) = (q_+, q_-) = 1$ .

Notice that these conditions have the exceptional solution,  $p = q_+ = 1$ ,  $q_- = 0$ . For this solution,  $\xi$  coincides with  $\eta_+$ , but in all other cases (except for the mirror image case  $\xi = \eta_-$ ) all three vectors point in distinct directions. This exceptional solution will also be exceptional in its thermodynamic properties. We don’t consider this exceptional case in this article.

From (5.8), we find

$$\beta_H = 4\pi n \frac{q_+ + q_-}{p} , \quad (5.9)$$

$$\Omega_H = \frac{1}{2n} \frac{q_+ - q_-}{q_+ + q_-} , \quad (5.10)$$

and therefore

$$\tilde{\Omega}_H = \beta_H\Omega_H = 2\pi \frac{q_+ - q_-}{p} . \quad (5.11)$$

We emphasize that the parameters  $p$ ,  $q_{\pm}$  are integers and therefore not continuously variable. For this reason — and in sharp contrast to instantons without NUT charge — the solutions considered here fall into disconnected families between which no continuous transition is possible. As a result, for a fixed set of  $p, q_+$  and  $q_-$ , the Euclidean metric has only one single continuous parameter. We can proceed now and compute the conserved mass  $\mathcal{M}$  and angular momentum  $\mathcal{J}$ , and we get

$$\mathcal{M} = \frac{2}{q_+ + q_-} m , \quad (5.12)$$

and

$$\mathcal{J} = \frac{2}{q_+ + q_-} a m . \quad (5.13)$$

The total Euclidean action, expressed in terms of the rationalized dimensionless variables  $\hat{\beta} = \beta/4\pi n$ , and  $\hat{\Omega} = \tilde{\Omega}/2\pi$ , is given by

$$I_E = \frac{4\pi n^2}{p} \frac{1 + \hat{\beta}^2 - \hat{\Omega}^2}{\hat{\beta} + \sqrt{(1 - \hat{\Omega}^2)(\hat{\beta}^2 - \hat{\Omega}^2)}} . \quad (5.14)$$

The entropy could be obtained from

$$S = \beta \mathcal{M} - \tilde{\Omega} \mathcal{J} - I_E , \quad (5.15)$$

where we have taken the thermodynamic quantities as the conserved quantities and analytically continue  $\hat{\Omega}$  to  $-i\hat{\Omega}$  (see more details in [21]) and is given by

$$S = \frac{4\pi n^2}{p\hat{\beta}} \frac{1 + \hat{\beta}^2 + \hat{\Omega}^2}{\hat{\beta} + \sqrt{(1 + \hat{\Omega}^2)(\hat{\beta}^2 + \hat{\Omega}^2)}} (2\hat{\beta} - \sqrt{\frac{\hat{\beta}^2 + \hat{\Omega}^2}{1 + \hat{\Omega}^2}}) . \quad (5.16)$$

The entropy (5.16) is definitely positive for all allowed values of  $\hat{\beta}$  and  $\hat{\Omega}$ . In fact one can show that  $0 \leq \hat{\Omega}^2 \leq 1 \leq \hat{\beta}$  with  $\hat{\Omega} = 1$  only if  $\hat{\beta} = 1$ .

## 6 Concluding Remarks

In this paper, we considered the class of Kerr-Bolt spacetimes without removing their singularities and showed that by using recently Kerr/CFT correspondence, the CFT could detect the presence of singularities. We found explicitly the near-horizon metric of the extremal spacetimes by taking the near-horizon procedure. The near-horizon geometry has topology of warped  $\text{AdS}_3$ . By choosing the proper boundary condition, we can find the diffeomorphism that generates a Virasoro algebra without any central charge. The generator of diffeomorphism which is a conserved charge can be used to construct an algebra under Dirac brackets.

This algebra is the same as diffeomorphism algebra but just with an extra central term. The central charge of the Virasoro algebra together with Frolov-Thorne temperature enable us to find the microscopic entropy of the extremal spacetime in dual chiral CFT. We showed the microscopic entropy doesn't satisfy the microscopic first law of thermodynamics unless the NUT charge of spacetime vanishes. This in turn show the metric (2.1) without removing its conical and Misner-Dirac singularities (given by equation (2.8)), leads to violation of the microscopic first law in CFT side. Our work provides further supportive evidence in favor of a Kerr/CFT correspondence.

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